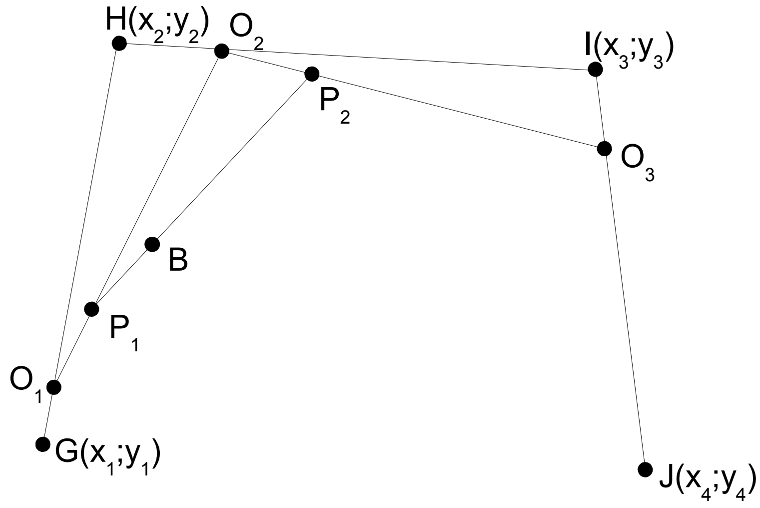

Quadratic Bezier curve is defined as a path of point moving along distance between two points, with double nesting, like on a picture. For c=1 each of moving points reaches destination. For c=0 points remain in starting point. When each starting and ending point is defined by set of variables x_1,y_1 and x_2,y_2, we can, by using proportion, find point on the way between start and the end. We just proportionally scale vector x_2x_1 attached at x_1:

$$x_c=x_1+c(x_2-x_1)$$
$$y_c=y_1+c(y_2-y_1)$$

c is turning x_c from x_1 into x_2. For c=1 x_c equals $x_1+x_2-x_1$. While x_1 reduces, x_c turns into x_2. For zero x_c equals x_1. Function of c is linear, we've got proportional and smooth change from x_1 to x_2, as long as c changes from 0 to 1.
Using this rule for points on the path from G to H, H to I, I to J, we define points O_1, O_2, O_3, using proportion written above, for each coordinate, according to formulas below:

$$O_1=\left(x_1+c(x_2-x_1);y_1+c(y_2-y_1)\right)$$
$$O_2=\left(x_2+c(x_3-x_2);y_2+c(y_3-y_2)\right)$$
$$O_3=\left(x_3+c(x_4-x_3);y_3+c(y_4-y_3)\right)$$

Then, along distances between these points another set of nested points slides. Let's name it P_1 and P_2. Formulas for x and y of these points, use the same proportion as above, with coordinates of O points taken as a variables, instead of coordinates of G, H, I, J:

$$P_1=\left([x_1+c(x_2-x_1)]+c\left([x_2+c(x_3-x_2)]-[x_1+c(x_2-x_1)]\right);[y_1+c(y_2-y_1)]+c\left([y_2+c(y_3-y_2)]-[y_1+c(y_2-y_1)]\right)\right)$$
$$P_2=\left([x_2+c(x_3-x_2)]+c\left([x_3+c(x_4-x_3)]-[x_2+c(x_3-x_2)]\right);[y_2+c(y_3-y_2)]+c\left([y_3+c(y_4-y_3)]-[y_2+c(y_3-y_2)]\right)\right)$$

Finally, along path between P_1 and P_2, slides final point B. Coordinates of this point are one more nesting in our equation of proportion. This time we use x and y of point P to create B:

$$B_x=[x_1+c(x_2-x_1)]+c\left([x_2+c(x_3-x_2)]-[x_1+c(x_2-x_1)]\right)+c\Big[\big[[x_2+c(x_3-x_2)]+c\left([x_3+c(x_4-x_3)]-[x_2+c(x_3-x_2)]\right)\big]-\big[[x_1+c(x_2-x_1)]+c\left([x_2+c(x_3-x_2)]-[x_1+c(x_2-x_1)]\right)\big]\Big]$$
$$B_y=[y_1+c(y_2-y_1)]+c\left([y_2+c(y_3-y_2)]-[y_1+c(y_2-y_1)]\right)+c\Big[\big[[y_2+c(y_3-y_2)]+c\left([y_3+c(y_4-y_3)]-[y_2+c(y_3-y_2)]\right)\big]-\big[[y_1+c(y_2-y_1)]+c\left([y_2+c(y_3-y_2)]-[y_1+c(y_2-y_1)]\right)\big]\Big]$$

Let's do the physical work and reduce this bulky equation a bit:

$$\left[x_1+c(x_2-x_1)\right]+c\left(\left[x_2+c(x_3-x_2)\right]-\left[x_1+c(x_2-x_1)\right]\right)+c\left[\left[\left[x_2+c(x_3-x_2)\right]+c\left(\left[x_3+c(x_4-x_3)\right]-\left[x_2+c(x_3-x_2)\right]\right)\right]-\left[\left[x_1+c(x_2-x_1)\right]+c\left(\left[x_2+c(x_3-x_2)\right]-\left[x_1+c(x_2-x_1)\right]\right)\right]\right]$$

$$x_1+c(x_2-x_1)+c\left(x_2+c(x_3-x_2)-x_1-c(x_2-x_1)\right)+c\left[x_2+c(x_3-x_2)+c\left(x_3+c(x_4-x_3)-x_2+c(x_3-x_2)\right)-x_1-c(x_2-x_1)-c\left(x_2+c(x_3-x_2)-x_1+c(x_2-x_1)\right)\right]$$

$$x_1+cx_2-cx_1+cx_2+c^2x_3-c^2x_2-cx_1-c^2x_2+c^2x_1+c\left[x_2+cx_3-cx_2+cx_3+c^2x_4-c^2x_3-cx_2-c^2x_3+c^2x_2-x_1-cx_2+cx_1-cx_2-c^2x_3+c^2x_2+cx_1+c^2x_2-c^2x_1\right]$$

$$x_1+cx_2-cx_1+cx_2+c^2x_3-c^2x_2-cx_1-c^2x_2+c^2x_1+cx_2+c^2x_3-c^2x_2+c^2x_3+c^3x_4-c^3x_3-c^2x_2-c^3x_3+c^3x_2-cx_1-c^2x_2+c^2x_1-c^2x_2-c^3x_3+c^3x_2+c^2x_1+c^3x_2-c^3x_1$$

$$\left[x_1-cx_1-cx_1+c^2x_1-cx_1+c^2x_1+c^2x_1-c^3x_1\right]+\left[cx_2+cx_2-c^2x_2-c^2x_2+cx_2-c^2x_2-c^2x_2+c^3x_2-c^2x_2-c^2x_2+c^3x_2+c^3x_2\right]+\left[c^2x_3+c^2x_3+c^2x_3-c^3x_3-c^3x_3-c^3x_3\right]+\left[c^3x_4\right]$$

$$x_1(1-3c+3c^2-c^3)+x_2(3c-6c^2+3c^3)+x_3(3c^2-3c^3)+x_4c^3$$

$$X_B=x_1(1-3c+3c^2-c^3)+3x_2(c-2c^2+c^3)+3x_3(c^2-c^3)+x_4c^3$$
$$Y_B=y_1(1-3c+3c^2-c^3)+3y_2(c-2c^2+c^3)+3y_3(c^2-c^3)+y_4c^3$$

Coordinates X_B and Y_B in function of c, varying from 0 to 1 draws quadratic Bezier curve. For lesser degrees, use points P as a entry point. To get further degrees of curves nest coordinates furthermore.

To develop velocity and acceleration of a curve use equations below:

$$\frac{\partial X_B}{\partial c}=3x_1(-1+2c-c^2)+3x_2(1-4c+3c^2)+3x_3(2c-3c^2)+3x_4c^2$$

$$\frac{\partial Y_B}{\partial c}=3y_1(-1+2c-c^2)+3y_2(1-4c+3c^2)+3y_3(2c-3c^2)+3y_4c^2$$

$$\frac{\partial^2 X_B}{\partial c}=6x_1(1-c)+6x_2(-2+3c)+6x_3(1-3c)+6x_4c$$

$$\frac{\partial^2 Y_B}{\partial c}=6y_1(1-c)+6y_2(-2+3c)+6y_3(1-3c)+6y_4c$$

From equation of differential calculated for c=0 and c=1, we can get starting and final coordinates of vectors of “handles”, that are usually used for drawing Bezier curve. Having equations of such, we can develop points H and I of a curve, for starting and ending points and lengths and directions of “handles”.

$$X_S=-3x_1+3x_2$$
$$Y_S=-3y_1+3y_2$$

$$X_E=-3x_3+3x_4$$
$$Y_E=-3y_3+3y_4$$

$$x_2=(X_S+3x_1)/3$$
$$y_2=(Y_S+3y_1)/3$$
$$x_3=(3x_4-X_E)/3$$
$$y_3=(3y_4-Y_E)/3$$

x_2, y_2, x_3, y_3 are missing points when points G and J are given with vectors curve velocity vectors X_S, Y_S, X_E, Y_E. X_S and Y_S are coordinates of vector of velocity of Bezier curve at start point x_1, y_1.

X_E and Y_E are coordinates of vector of velocity of Bezier curve at end point x_4, y_4.

When all points are found, use formula for point B to draw quadratic Bezier curve.